

Trigonometric Identities

Reciprocal

$$\begin{aligned} \sec x &= \frac{1}{\cos x} & \csc x &= \frac{1}{\sin x} \\ \tan x &= \frac{\sin x}{\cos x} & \cot x &= \frac{\cos x}{\sin x} \\ \cot x &= \frac{1}{\tan x} & \tan x &= \frac{1}{\cot x} \end{aligned}$$

Pythagorean

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x \\ 1 + \cot^2 x &= \csc^2 x \end{aligned}$$

Negative Angle

$$\begin{aligned} \sin(-x) &= -\sin x \\ \cos(-x) &= \cos x \\ \tan(-x) &= -\tan x \end{aligned}$$

Addition and Subtraction

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Periodicity

$$\sin(x + 2\pi) = \sin x \quad \cos(x + 2\pi) = \cos x$$

$$\csc(x + 2\pi) = \csc x \quad \sec(x + 2\pi) = \sec x$$

$$\tan(x + \pi) = \tan x \quad \cot(x + \pi) = \cot x$$

Cofunction

$$\sin x = \cos\left(\frac{\pi}{2} - x\right) \quad \cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\tan x = \cot\left(\frac{\pi}{2} - x\right) \quad \cot x = \tan\left(\frac{\pi}{2} - x\right)$$

$$\sec x = \csc\left(\frac{\pi}{2} - x\right) \quad \csc x = \sec\left(\frac{\pi}{2} - x\right)$$

Product

$$\sin x \cos y = \frac{1}{2} (\sin(x + y) + \sin(x - y))$$

$$\sin x \sin y = \frac{1}{2} (\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2} (\cos(x + y) + \cos(x - y))$$

$$\cos x \sin y = \frac{1}{2} (\sin(x + y) - \sin(x - y))$$

Factoring

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

Double Angle

$$\sin(2x) = 2 \sin x \cos x \quad \cos(2x) = 1 - 2 \sin^2 x$$

$$\cos(2x) = \cos^2 x - \sin^2 x \quad \tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cos(2x) = 2 \cos^2 x - 1$$

Half-Angle

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \quad \tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}} \quad \tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}$$